ISOP PYP

MATHEMATICS

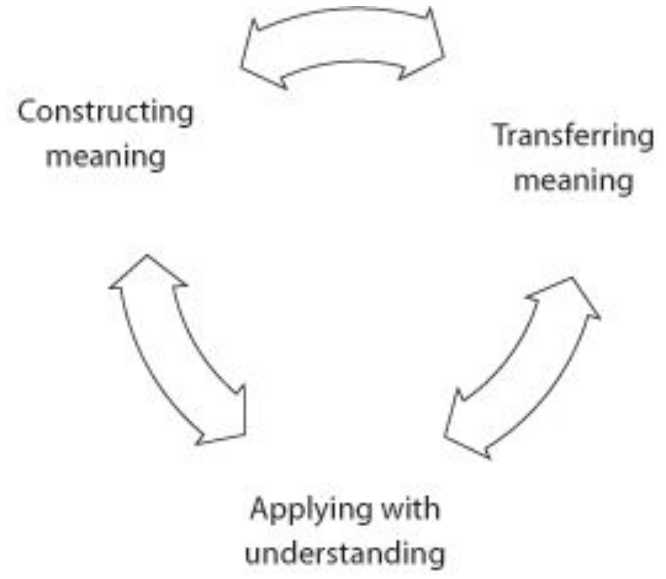
SCOPE AND SEQUENCE

The ISoP PYP Mathematics Scope and Sequence has been developed on the basis of:
- IB PYP Mathematics scope and sequence
- The Polish National Curriculum for Mathematics
- The Australian Curriculum for Mathematics (ACARA)
- Cambridge Primary Mathematics Curriculum Framework

MATHEMATICS IN THE PYP

What the PYP believes about learning mathematics	The power of mathematics for describing and analysing the world around us is such that it has become a highly effective tool for solving problems. It is also recognized that students can appreciate the intrinsic fascination of mathematics and explore the world through its unique perceptions. In the same way that students describe themselves as "authors" or "artists", a school's programme should also provide students with the opportunity to see themselves as "mathematicians", where they enjoy and are enthusiastic when exploring and learning about mathematics. In the IB Primary Years Programme (PYP), mathematics is also viewed as a vehicle to support inquiry, providing a global language through which we make sense of the world around us. It is intended that students become competent users of the language of mathematics, and can begin to use it as a way of thinking, as opposed to seeing it as a series of facts and equations to be memorized.
Mathematics in a transdisciplinary programme	Wherever possible, mathematics should be taught through the relevant, realistic context of the units of inquiry. The direct teaching of mathematics in a unit of inquiry may not always be feasible but, where appropriate, prior learning or follow-up activities may be useful to help students make connections between the different aspects of the curriculum. Students also need opportunities to identify and reflect on "big ideas" within and between the different strands of mathematics, the programme of inquiry and other subject areas. Links to the transdisciplinary themes should be explicitly made, whether or not the mathematics is being taught within the programme of inquiry. A developing understanding of these links will contribute to the students' understanding of mathematics in the world and to their understanding of the transdisciplinary theme. The role of inquiry in mathematics is important, regardless of whether it is being taught inside or outside the programme of inquiry. However, it should also be recognized that there are occasions when it is preferable for students to be given a series of strategies for learning mathematical skills in order to progress in their mathematical understanding rather than struggling to proceed.
How children learn mathematics	It is important that learners acquire mathematical understanding by constructing their own meaning through ever-increasing levels of abstraction, starting with exploring their own personal experiences, understandings and knowledge. Additionally, it is fundamental to the philosophy of the PYP that, since it is to be used in real-life situations, mathematics needs to be taught in relevant, realistic contexts, rather than by attempting to impart a fixed body of knowledge directly to students

How children learn mathematics can be described using the following stages:



Constructing meaning about mathematics

Learners construct meaning based on their previous experiences and understanding, and by reflecting upon their interactions with objects and ideas. Therefore, involving learners in an active learning process, where they are provided with possibilities to interact with manipulatives and to engage in conversations with others, is paramount to this stage of learning mathematics.

When making sense of new ideas all learners either interpret these ideas to conform to their present understanding or they generate a new understanding that accounts for what they perceive to be occurring. This construct will continue to evolve as learners experience new situations and ideas, have an opportunity to reflect on their understandings and make connections about their learning.

Transferring meaning into symbols

Only when learners have constructed their ideas about a mathematical concept should they attempt to transfer this understanding into symbols. Symbolic notation can take the form of pictures, diagrams, modelling with concrete objects and mathematical notation. Learners should be given the opportunity to describe their understanding using their own method of symbolic notation, then learning to transfer them into conventional mathematical notation.

Applying with understanding

Applying with understanding can be viewed as the learners demonstrating and acting on their understanding. Through authentic activities, learners should independently select and use appropriate symbolic notation to process and record their thinking. These authentic activities should include a range of practical hands-on problem-solving activities and realistic situations that provide the opportunity to demonstrate mathematical thinking through presented or recorded formats. In this way, learners are able to apply their understanding of mathematical concepts as well as utilize mathematical skills and knowledge.

As they work through these stages of learning, students and teachers use certain processes of mathematical reasoning.

- They use patterns and relationships to analyse the problem situations upon which they are working.
- They make and evaluate their own and each other's ideas.
- They use models, facts, properties and relationships to explain their thinking.
- They justify their answers and the processes by which they arrive at solutions.

In this way, students validate the meaning they construct from their experiences with mathematical situations. By explaining their ideas, theories and results, both orally and in writing, they invite constructive feedback and also lay out alternative models of thinking for the class. Consequently, all benefit from this interactive process.

STRANDS OF MATHEMATICS IN THE PYP

Data handling	Data handling allows us to make a summary of what we know about the world and to make inferences about what we do not know. • Data can be collected, organized, represented and summarized in a variety of ways to highlight similarities, differences and trends; the chosen format should illustrate the information without bias or distortion. • Probability can be expressed qualitatively by using terms such as “unlikely”, “certain” or “impossible”. It can be expressed quantitatively on a numerical scale.
Measurement	To measure is to attach a number to a quantity using a chosen unit. Since the attributes being measured are continuous, ways must be found to deal with quantities that fall between numbers. It is important to know how accurate a measurement needs to be or can ever be.
Shape and space	The regions, paths and boundaries of natural space can be described by shape. An understanding of the interrelationships of shape allows us to interpret, understand and appreciate our two-dimensional (2D) and three-dimensional (3D) world.
Pattern and function	To identify pattern is to begin to understand how mathematics applies to the world in which we live. The repetitive features of patterns can be identified and described as generalized rules called “functions”. This builds a foundation for the later study of algebra.
Number	Our number system is a language for describing quantities and the relationships between quantities. For example, the value attributed to a digit depends on its place within a base system. Numbers are used to interpret information, make decisions and solve problems. For example, the operations of addition, subtraction, multiplication and division are related to one another and are used to process information in order to solve problems. The degree of precision needed in calculating depends on how the result will be used.

Grade K1 learning outcomes

Data handling	Measurement	Shape and space	Pattern and function
Conceptual understandings: Organizing objects and events helps us to solve problems (P1) We collect information to make sense of the world around us (P1) **Learners will develop an understanding that:** - sets can be organized by different attributes - information about themselves and their surrounding can be obtained in different ways	**Conceptual understandings:** Events can be ordered and sequenced (P1) Measurement involves comparing objects and events (P1) **Learners will develop an understanding that:** - events in daily routines can be described and sequenced, for example, before, after, bedtime, storytime, today, tomorrow - attributes of real objects can be compared and described, for example, longer, shorter, heavier, empty, full, hotter, colder	**Conceptual understandings:** Shapes can be described and organized according to their properties (P1) Objects in our immediate environment have a position in space that can be described according to a point of reference (P1) **Learners will develop an understanding that:** - 2D and 3D shapes have characteristics that can be described and compared - common language can be used to describe position and direction, for example, inside, outside, above, below, next to, behind, in front of, up, down	**Conceptual understandings:** Patterns and sequences occur in everyday situations (P1) **Learners will develop an understanding that:** - patterns can be found in everyday situations, for example, sounds, actions, objects, nature
1. Match pairs of identical or related objects 2. Select criteria for sorting 3. Sort and label real objects into sets by attributes (colour, type, size, shape) 4. Answer yes/no questions about themselves and familiar objects to collect information 5. Representing information using simple displays (creating living graphs using real objects and people)	1. Introduce the language of length (long/short), mass (heavy/light) and capacity (full/empty, more/less) 2. Identify, compare and describe attributes of real objects, for example, longer/shorter, heavier/lighter, full/empty 3. Put sets of real objects in order of size and length 4. Identify, describe and sequence events in their daily school routine, for example, after, before, snack, lunch 5. Begin to read o'clock time (snack - 9 o'clock, lunch - 12 o'clock, etc.) 6. List days of the week, months of the year and seasons in order 7. Connect days of the week to familiar events and actions	1. Sort, describe and name familiar 2D shapes (circle, square, rectangle, triangle, diamond, heart, oval) 2. Create patterns using familiar 2D shapes 3. Use everyday language to describe position and direction (left, right, behind, next to) 4. Explore and describe the paths, regions and boundaries of their immediate environment	1. Recognize and describe simple patterns (sounds, actions, objects, nature) 2. Create simple patterns using real objects and drawings

Grade K1 learning outcomes

Number
Conceptual understandings: Numbers are naming systems (P1) Numbers can be used in many ways for different purposes in the real world (P1) Numbers are connected to each other through a variety of relationships (P1) Making connections between our experiences with numbers can help us to develop number sense (P1) **Learners will develop an understanding that:** - one-to-one correspondence - for a set of objects, the number name of the last object counted describes the quantity of the whole set - numbers can be constructed in multiple ways, for example, by combining and partitioning - conservation of number - the relative magnitude of whole numbers
1. Read, write and model numbers to 10 2. Count to 10 forwards and backwards by naming numbers in sequences from any starting point 3. Count to determine the number of objects in a set (up to 10 objects; one-to-one correspondence) 4. Explore the conservation of number through the use of manipulatives (regardless of the arrangement, the amount stays the same) 5. Subitise small collections of objects (up to 5 objects without counting) 6. Compare two numbers using more and less 7. Order a set of selected numbers 8. Connect number names and numerals up to 10 to the quantities they represent, including zero 9. Use ordinal numbers to 10 in real-life situations 10. Introduce vocabulary involved in adding (one more, add, and) 11. Introduce vocabulary involved in subtracting (one less/take away) 12. Represent practical situations to model addition and subtraction within 10 on concrete materials or pictures

K2 learning outcomes

Data handling	Measurement	Shape and space	Pattern and function
Conceptual understandings: Events in daily life involve chance (P1) Organizing objects and events helps us to solve problems (P1) We collect information to make sense of the world around us (P1) **Learners will develop an understanding that:** - sets can be organized by different attributes - information about themselves and their surrounding can be obtained in different ways	**Conceptual understandings:** Events can be ordered and sequenced (P1) Measurement involves comparing objects and events (P1) Objects have attributes that can be measured using non-standard units (P1) **Learners will develop an understanding that:** - events in daily routines can be described and sequenced, for example, before, after, bedtime, storytime, today, tomorrow - attributes of real objects can be compared and described, for example, longer, shorter, heavier, empty, full, hotter, colder	**Conceptual understandings:** Shapes can be described and organized according to their properties (P1) Objects in our immediate environment have a position in space that can be described according to a point of reference (P1) **Learners will develop an understanding that:** - 2D and 3D shapes have characteristics that can be described and compared - common language can be used to describe position and direction, for example, inside, outside, above, below, next to, behind, in front of, up, down	**Conceptual understandings:** Patterns and sequences occur in everyday situations (P1) Patterns repeat and grow (P1) Patterns can be represented using numbers and other symbols (P2) **Learners will develop an understanding that:** - patterns can be found in everyday situations, for example, sounds, actions, objects, nature - patterns can be found in numbers, for example, odd and even numbers, skip counting
1. Identify outcomes of familiar events that involve chance (certain, impossible events) and describe them using everyday language 2. Sort, describe and label real objects and events into sets by attributes 3. Choose simple questions and gather appropriate responses for simple investigations 4. Collect information and represent them through real objects or drawings where one object or drawing represents one data value 5. Describe data displays (identifying categories with the greatest or least number of objects, comparing quantities using number words)	1. Identify, measure and compare the length, mass and capacity of pairs of objects using non-standard units 2. Use non-standard units of measurement to solve problems in real-life situations involving length, mass and capacity 3. Identify, describe and sequence events in their daily routine, for example, after, before, tomorrow, today, bedtime, storytime 4. Describe duration using months, weeks, days and hours 5. Tell time to the full hour on analogue and digital clocks 6. List days of the week, months of the year and seasons in order	1. Sort, describe and compare familiar 2D shapes (square, diamond, circle, triangle, rectangle, oval) 2. Sort, describe and compare familiar 3D objects (ball shape, box shape) using obvious features (corners, edges, faces) 3. Create simple shape patterns 4. Create simple symmetrical drawings 5. Describe the position of an object in relation to another object (between, on top, inside, beside, under, in front of, behind, outside, before, after, near, far, up, down, next to) 6. Explore and describe the paths, regions and boundaries of their immediate environment	1. Recognise, describe, extend and create patterns using real objects and drawings 2. Describe, extend and create number patterns that increase and decrease 3. Describe, extend and create number patterns formed by skip counting by twos

K2 learning outcomes

Number
Conceptual understandings: Numbers are naming systems (P1) Numbers can be used in many ways for different purposes in the real world (P1) Numbers are connected to each other through a variety of relationships (P1) Making connections between our experiences with numbers can help us to develop number sense (P1) **Learners will develop an understanding that:** - one-to-one correspondence - for a set of objects, the number name of the last object counted describes the quantity of the whole set - numbers can co constructed in multiple ways, for example, by combining and partitioning - conservation of number - the relative magnitude of whole numbers - whole-part relationships
1. Read, write and model numbers to 20 2. Read and write number names to 10 3. Count forwards and backwards in 1s from any starting point and in 2s from zero to 20 4. Count to determine the number of objects in a set (one-to-one correspondence) 5. Locate numbers to 20 on a number line 6. Subitise small collections of objects (up to 5 objects without counting) 7. Compare and order numbers to 20 8. Connect number names and numerals up to 20 to the quantities they represent, including zero 9. Use ordinal numbers to 10 to describe the position in a sequence 10. Estimate quantities to 10 and count to check 11. Use mathematical vocabulary and symbols of addition and subtraction (add, take away, +, -, =) 12. Model addition facts for 10 (making 10) using pictures and concrete materials 13. Represent and solve simple addition and subtraction problems at least within 10 using a range of strategies (count on, count back) 14. Model equal groups 15. Share collections of objects (into 2 and 3 sets) 16. Recognise and describe one-half as one of two equal parts of a whole 17. Recognise Polish coins 18. Solve simple addition and subtraction problems involving money

Grade 1 learning outcomes

Data handling	Measurement
Conceptual understandings: Events in daily life involve chance (P1) Objects and events can be organized in different ways (P2) Information can be expressed as organised and structured data (P2) **Learners will develop an understanding that:** - sets can be organized by one or more attributes - information about themselves and their surrounding can be collected and recorded in different ways	**Conceptual understandings:** Objects have attributes that can be measured using non-standard units (P1) Standard units allow us to have a common language to identify, compare, order and sequence objects and events (P2) We use tools to measure the attributes of objects and events (P2) **Learners will develop an understanding that:** - tools can be used to measure - calendars can be used to determine the date, and to identify and sequence days of the week and months of the year - time is measured using universal units of measure, for example, years, months, days, hours, minutes and seconds
1. Identify chance in daily events (certain/possible, impossible events) 2. Describe outcomes of events as likely and unlikely 3. Sort, describe and label real objects into sets by one or more attributes 4. Discuss data represented in tree, Venn and Carroll diagrams 5. Identify a question of interest based on one categorical variable and gather data relevant to the question 6. Collect, classify and represent data using lists, tables, pictographs, tally marks as well as simple bar graphs from a graph of real objects 7. Interpret data for the purpose of answering questions related to comparing quantities (more, fewer, less than, greater than)	1. Compare and order several shapes and objects based on length, area and capacity using appropriate non-standard units 2. Compare masses of objects using balance scales 3. Use non-standard units of measurement to solve problems in real-life situations involving length, area, mass and capacity 4. Estimate and compare lengths of time: minute, hour, day, week, month and year 5. Read and write analogue and digital time to the full hour and half hour, using the language of 'past' 6. Use a calendar to determine the date, and to identify and sequence days of the week and months of the year including determining the number of days in each month

Grade 1 learning outcomes

Shape and space	Pattern and function
Conceptual understandings: Shapes are classified and named according to their properties (P2) Specific vocabulary can be used to describe an object's position in space (P2) **Learners will develop an understanding that:** - there are relationships among and between 2D and 3D shapes - examples of symmetry and transformations can be found in their immediate environment - directions can be used to describe pathways, regions, positions and boundaries of their immediate environment	**Conceptual understandings:** Patterns and sequences occur in everyday situations (P1) Patterns repeat and grow (P1) Patterns can be represented using numbers and other symbols (P2) **Learners will develop an understanding that:** - patterns can be found in everyday situations, for example, sounds, actions, objects, nature - patterns can be found in numbers, for example, odd and even numbers, skip counting
1. Identify, name and draw 2D shapes (square, circle, triangle, rectangle, hexagon, pentagon, oval) 2. Describe the features of 2D shapes (sides, corners) 3. Identify, name and sort 3D shapes (cone, cube, cylinder, sphere) 4. Describe the features of 3D objects (faces, corners, edges) 5. Make models of three-dimensional objects 6. Analyse and describe the relationship between 2D and 3D shapes 7. Find and explain symmetry in the environment 8. Create and describe symmetrical designs 9. Describe the position of an object in relation to another object (beside, in front of, up. down, next to, behind, between, below, above) 10. Give and follow simple directions describing paths, regions, positions and boundaries of their immediate environment	1. Describe, extend and create various patterns 2. Describe, extend and create number patterns, for example, odd and even numbers, skip counting 3. Identify missing elements in simple number patterns 4. Identify the commutative property of addition (4+3=3+4) 5. Solve word problems by using number sentences for addition or subtraction

Grade 1 learning outcomes

Number
Conceptual understandings: The base 10 place value system is used to represent numbers and number relationships (P2) Fractions are ways of representing whole-part relationships (P2) The operations of addition, subtraction, multiplication and division are related to each other and are used to process information to solve problems (P2-3) Number operations can be modelled in a variety of ways (P2) There are many mental methods that can be applied for exact and approximate computations (P2)
1. Read, write and model numbers to 100 2. Read and write number names to 100 (78 - seventy-eight) 3. Count to 100 forwards and backwards in 1s, 2s, 3s, 5s and 10s from any starting point 4. Compare and order numbers to at least 100 5. Apply place value to partition collections up to 100 in tens and ones/units 6. Identify odd and even numbers 7. Read, write, compare and order ordinal numbers to 31 (including the use of them in dates) 8. Estimate quantities to 20 and count to check 9. Use mathematical vocabulary and symbols of addition and subtraction (+, -, =, add, sum, total, subtract, take away, difference) 10. Automatically recall addition and subtraction number facts to 10 11. Represent and solve addition and subtraction problems at least within 20 using a range of efficient mental and written strategies (count on, count back, doubles, number line) 12. Represent multiplication as repeated addition, groups and arrays 13. Represent division as grouping into equal sets 14. Recognise and interpret common uses of halves and quarters of shapes and collections 15. Recognise, describe and order Polish coins and notes according to their value 16. Count small collections of coins and notes

Grade 2 learning outcomes

Data handling	Measurement
Conceptual understandings: Some events in daily life are more likely to happen than the others (P2) Objects and events can be organized in different ways (P2) Information can be expressed as organised and structured data (P2) **Learners will develop an understanding that:** - the concept of chance in daily events - sets can be organized by one or more attributes - information about themselves and their surrounding can be collected and recorded in different ways	**Conceptual understandings:** Objects have attributes that can be measured using non-standard units (P1) Standard units allow us to have a common language to identify, compare, order and sequence objects and events (P2) We use tools to measure the attributes of objects and events (P2) Estimation allows us to measure with different levels of accuracy (P2) **Learners will develop an understanding that:** - tools can be used to measure - the use of standard units to measure, for example, length, mass, money, time, temperature - calendars can be used to determine the date, and to identify and sequence days of the week and months of the year
1. Identify daily events that involve chance 2. Classify events using the language of chance (impossible, less likely/unlikely, maybe, most likely/likely, certain) 3. Conduct simple chance experiments (e.g. tossing a coin), identify and describe their possible outcomes 4. Sort, describe and label real objects and events into sets by one or more attributes 5. Discuss the relationship between sets of objects and events represented in tree, Venn and Carroll diagrams and create simple diagrams 6. Identify questions or issues for categorical variables. Identify data sources and plan methods of data collection and recording 7. Collect data, organise them into categories and create displays using tally marks, lists, tables, pictographs, simple bar graphs 8. Interpret and compare data displays describing their similarities and differences	1. Estimate, measure, compare and record lengths and distances using non-standard and standard units (m, cm) 2. Estimate, measure, compare and record masses and capacities of two or more objects using non-standard units 3. Estimate, measure, compare and record areas using non-standard units 4. Estimate, compare and record temperatures using non-standard and standard (^{0}C) units 5. Use standard and non-standard units of measurement to solve problems in real-life situations involving length, area, mass, capacity and temperature 6. Estimate and compare lengths of time: second, minute, hour, day, week, month and year, and investigate the relationship between them 7. Read and write digital and analogue time to the full hour, half hour and quarter hour using the language of ‘past’ and ‘to’ 8. Use a calendar to determine date, and to identify the year, month, week and day 9. Use familiar measures of time to assist with problem solving in real-life situations 10. Use timelines in real-life situations

Grade 2 learning outcomes

Shape and space	Pattern and function
Conceptual understandings: Shapes are classified and named according to their properties (P2) Some shapes are made up of parts that repeat in some way (P2) Specific vocabulary can be used to describe an object's position in space (P2) **Learners will develop an understanding that:** - there are relationships among and between 2D and 3D shapes - 2D and 3D shapes can be created by putting together and/or taking apart other shapes - examples of symmetry and transformations can be found in their immediate environment - directions can be used to describe pathways, regions, positions and boundaries of their immediate environment	**Conceptual understandings:** Whole numbers exhibit patterns and relationships that can be observed and described (P2) Patterns can be represented using numbers and other symbols (P2) **Learners will develop an understanding that:** - patterns can be found in numbers, for example, odd and even numbers, skip counting - the inverse relationship between addition and subtraction - the associative and commutative properties of addition - the inverse relationship between multiplication and division
1. Identify, draw and describe key features of 2D shapes (square, circle, triangle, rectangle, pentagon, hexagon, trapezium, rhombus) 2. Identify parallel lines 3. Recognise 3D shapes (cone, cube, cylinder, sphere, prisms) and describe their key features (faces, corners, edges, curved surfaces) 4. Draw 3D objects from the top, front and side view 5. Analyse and describe the relationship between 2D and 3D shapes 6. Identify lines of reflective symmetry and create symmetrical patterns 7. Create and describe patterns with the use of transformations such as flip, slide and turn 8. Identify angles as measures of turn and compare angle sizes in everyday situations (clock arme, door) 9. Identify right angles 10. Distinguish between right angles and angles smaller and larger than a right angle 11. Describe the position of an object 12. Create and follow directions to draw a path on a simple plan to show a described route (up, down, left, right)	1. Describe, extend and create a variety of number patterns resulting from performing addition and subtraction including supplying missing elements 2. Identify and describe the rules for number patterns 3. Identify and model the inverse relationship between addition and subtraction (3+4=7, 7-3=4) - check addition using subtraction and check subtraction using addition 4. Identify the properties of addition: commutative 4+3=3+4 and associative 2+(5+3)=(2+5)+3 - check addition by adding in a different order 5. Identify and model the relationship between multiplication and addition (repeated addition) 6. Identify and apply the commutative property of multiplication (3x2=2x3) - check multiplication by revising the order, e.g. 6x4=24 by doing 4x6 7. Identify and model the inverse relationship between multiplication and division - check division using multiplication and check multiplication using division 8. Use number patterns and relationships to solve problems 9. Solve word problems by using number sentences for addition or subtraction (single-step word problems)

Grade 2 learning outcomes

Number
Conceptual understandings: The base 10 place value system is used to represent numbers and number relationships (P2) Fractions are ways of representing whole-part relationships (P2) The operations of addition, subtraction, multiplication and division are related to each other and are used to process information to solve problems (P2-3) Number operations can be modelled in a variety of ways (P2) There are many mental methods that can be applied for exact and approximate computations (P2)
1. Read and write numbers up to 1000 2. Write number names to 1000 (178 - one hundred and seventy-eight) 3. Count on and back in ones, tens and hundreds from two- and three-digit numbers 4. Count on and back in steps of 2, 3, 4 and 5 to at least 50 5. Apply place value to partition, rearrange and regroup numbers to 1000 into hundreds, tens and ones/units 6. Compare and order numbers to at least 1000 (<, >, =) - focus on two-digit and three-digit numbers 7. Investigate the conditions required for a number to be odd or even and identify odd and even numbers 8. Read, write, compare and order ordinal numbers to 100 9. Estimate quantities to 100 and count to check (group in tens) 10. Round two-digit numbers to the nearest 10 and three-digit numbers to the nearest 100 11. Interpret fractions as several parts of one whole, e.g. ¾ is three quarters and ⅔ is two thirds 12. Model and represent fractions including ½, ¼ and their multiples to a complete whole 13. Recognise simple mixed fractions in context, e.g. 1 ½, 2 ¼ 14. Order simple or mixed fractions on a number line (halves, quarters), e.g. using the knowledge that ½ comes half way between ¼ and ¾, and that 1 ½ comes half way between 1 and 2 15. Model and describe the division of a collection into halves and quarters (½ of 10, ¼ of 20) 16. Find halves, thirds, quarters and tenths of shapes and numbers (whole number answers) 17. Begin to relate fractions to division 18. Automatically recall addition and subtraction facts for all numbers to 20 19. Find complements to 100, solving number equations such as 78+_=100 20. Use written algorithm for addition and subtraction without trading (12+4, 15-3) 21. Represent and solve addition and subtraction problems at least within **200** using increasingly efficient mental and written strategies for computation and appropriate digital technologies (number line, hundred square, addition and subtraction patterns, doubles, near doubles) - add and subtract pairs of two-digit numbers; add and subtract single-digit numbers to/from three-digit numbers 22. Model and describe multiplication as repeated addition 23. Model and describe division as sharing into equal groups 24. Use mathematical vocabulary and symbols of multiplication and division (times, multiply, share equally, divide, x, ÷) 25. Recall quickly multiplication facts for 1x, 2x, 3x, 5x and 10x tables and related division facts

26. Begin to know 4x table
27. Recognise multiples of 2, 5 and 10 up to 1000
28. Understand that division can leave a remainder (initially as 'some left over')
29. Use a range of mental strategies and concrete materials for multiplication and division within 100 - multiply pairs of single-digit numbers; divide two-digit numbers by single-digit numbers (without remainders); multiply teens numbers by 3 and 5; begin to divide two-digit numbers just beyond 10x tables, e.g. 60 ÷ 5=12
30. Recognise, describe and order Polish (PLN) and European (EUR) coins and notes according to their value
31. Represent money values (PLN and EUR) in multiple ways and count the change required for simple transactions to the nearest five grosz or cents - podobne w G3

Mental strategies:

- Find 1, 10, 100 more/less than two- and three-digit numbers
- Find 20, 30, 40, 50, 60, 70, 80, 90, 100, 200, 300 more/less than three-digit numbers
- Derive quickly pairs of multiples of 100 with a total of 1000, e.g. 400+600=1000
- Derive quickly pairs of multiples of 5 with a total of 100, e.g. 45+55=100
- Derive quickly doubles of all whole numbers to 20 and derive related halves
- Derive quickly doubles of multiples of 5 to 100 and multiples of 50 to 500
- Add and subtract 10 and multiples of 10 to and from two- and three-digit numbers
- Add 100 and multiples of 100 to three-digit numbers
- Re-order addition to help with the calculation, e.g. 41+54, by adding 40 to 54, then 1
- Understand the effect of multiplying two-digit numbers by 10

Grade 3 learning outcomes

Data handling	
Conceptual understandings: Some events in daily life are more likely to happen than the others (P2) Probability can be based on experimental events in daily life (P3) Data can be collected, organized, displayed and analysed in different ways (P3)	**Learners will develop an understanding that:** - the concept of chance in daily events - probability is based on experimental events - data can be collected, displayed and interpreted using simple graphs, for example, bar graphs, line graphs - scale can represent different quantities in graphs - the mode can be used to summarize a set of data

Probability

1. Describe various events that involve chance
2. Classify events and order their chances of occurring using the language of chance (impossible, less likely/unlikely, maybe, most likely/likely, certain)
3. Conduct simple chance experiments, identify and describe their possible outcomes - 2 possible outcomes, e.g. tossing a coin

Organising, categorising and representing data

4. Identify, select and trial methods for data collection, including survey questions and recording sheets
5. Construct suitable data displays from given or collected data including diagrams (Venn and Carroll - two or three criteria), tally marks and frequency tables, pictographs (symbols representing 1, 2, 5, 10 or 20 units), bar graphs (intervals labelled in ones, twos, fives, tens or twenties) and simple line graphs
6. Find and interpret range (lowest and highest value) of a set of data

Grade 3 learning outcomes

Measurement	
Conceptual understandings: Standard units allow us to have a common language to identify, compare, order and sequence objects and events (P2) Estimation allows us to measure with different levels of accuracy (P2) Objects and events have attributes that can be measured using appropriate tools (P3) Relationships exist between standard units that measure the same attributes (P3)	**Learners will develop an understanding that:** - the use of standard units to measure, for example, length, mass, money, time, temperature - measures can fall between numbers on a measurement scale, for example, 3½ kg, between 4 and 5 cm - the relationship between units, for example, metres, centimetres and millimetres

Length, mass and capacity

1. Estimate, measure, compare and record lengths using standard units (m, cm, mm)
2. Estimate, measure, compare and record masses using standard units (kg, g)
3. Estimate, measure, compare and record capacities using standard units (l, ml)
4. Know and use the relationships between familiar units of length, mass and capacity, e.g. 1m=100cm, 1l=1000ml; know the meaning of 'kilo', 'centi', and 'milli'
5. Convert between common metric units of length (mm, cm and m) - no decimals
6. Describe measures that fall between numbers on a measurement scale, e.g., 2 ½ cm, between 2cm and 3cm
7. Use standard units of measurement and appropriate tools to solve problems in real-life situations involving length, mass and capacity

Area and perimeter

8. Measure and calculate the perimeter of rectangles
9. Understand that area is measured in square units (cm^2)
10. Find the area of rectilinear shapes drawn on a square grid by counting squares

Time

11. Read and tell time to the minute on 12-hour digital and analogue clocks
12. Use am and pm notation
13. Convert between common metric units of time (h, min)
14. Use familiar measures of time and tools (timetables, calendars) to assist with problem solving in real-life situations

Temperature

15. Estimate, measure, compare and record temperatures using standard units (^0C)

Grade 3 learning outcomes

Shape and space	
Conceptual understandings: Changing the position of a shape does not alter its properties (P3) Shapes can be transformed in different ways (P3) Geometric shapes and vocabulary are useful for representing and describing objects and events in real-world situations (P3)	**Learners will develop an understanding that:** - the common language used to describe shapes - the properties of regular and irregular polygons - an angle as a measure of rotation - directions for location can be represented by coordinates on a grid - geometric shapes are useful for representing real-world situations

Shape and geometric reasoning

1. Identify, name, draw and describe key features of regular and irregular polygons (triangles, a range of quadrilaterals including parallelograms and trapeziums, pentagon, hexagon, heptagon, octagon)
2. Classify polygons (including a range of quadrilaterals) using criteria such as the number of right angles, whether or not they are regular and their symmetrical properties
3. Identify parallel lines; recognise them in 2D shapes, drawings and the environment
4. Identify, name, sort, sketch and describe key features of prisms, pyramids, cones, cylinders and spheres
5. Connect 3D shapes with their 2D representations - drawings/sketches
6. Draw lines of reflective symmetry and create symmetrical patterns, pictures and shapes; find examples of symmetry in the environment an in art
7. Analyse angles by comparing and describing rotations (whole turn, half turn, quarter turn)
8. Know that angles are measured in degrees and that one whole turn is 360^{o}, half turn is 180^{o} and quarter turn is 90^{o}
9. Compare angles smaller than 180^{o} and classify them as equal to, greater than (blunt) or less than (sharp) a right angle

Position and movement

10. Create and describe patterns with the use of transformations such as flip, slide and turn
11. Describe the position of different objects and locate them on a grid using simple coordinates (rows and columns are numbered and/or lettered)
12. Use simple scales, keys and directions to interpret information contained in basic maps
13. Use N, S, E, W, NE, NW, SE, SW to describe locations

Grade 3 learning outcomes

Pattern and function	
Conceptual understandings: Functions are relationships or rules that uniquely associate members of one set with members of another set (P3) By analysing patterns and identifying rules for patterns it is possible to make predictions (P3)	**Learners will develop an understanding that:** - patterns can be analysed and rules identified - the inverse relationship between addition and subtraction - the associative and commutative properties of addition - multiplication is repeated addition and that division is repeated subtraction - the inverse relationship between multiplication and division - the associative and commutative properties of multiplication

1. Describe, extend and create number patterns resulting from performing addition, subtraction, multiplication and division
2. Describe and represent the rules for patterns in a variety of ways (using words and symbols)
3. Use equivalent number sentences involving addition and subtraction to find unknown quantities (23+...=57-9)
4. Use the inverse relationship between addition and subtraction to check the results (3+4=7, 7-3=4)
5. Use the properties of addition to find an easier way to add: commutative 4+3=3+4 and associative 2+(5+3)=(2+5)+3
6. Identify and model the relationship between multiplication and addition (repeated addition) and between division and subtraction (repeated subtraction)
7. Identify and model the inverse relationship between multiplication and division and use it to check results (12÷4=3, 3x4=12)
8. Identify the properties of multiplication: commutative 9x2=2x9 and associative 2x(3x1)=(2x3)x1 and use those properties to find the easier way to multiply
9. Use number patterns and relationships of the four operations to solve problems
10. Solve word problems by using number sentences involving addition, subtraction, multiplication or division where there is no remainder (single-step word problems)

Grade 3 learning outcomes

Number
Conceptual understandings: The base 10 place value system can be extended to represent magnitude (P3) Fractions and decimals are ways of representing whole-part relationships (P3) The operations of addition, subtraction, multiplication and division are related to each other and are used to process information to solve problems (P2-3) Even complex operations can be modelled in a variety of ways, for example, an algorithm is a way to represent operation (P3) There are many mental methods that can be applied for exact and approximate computations (P2)
1. Read and write numbers up to 10 000 2. Write the number names to 10 000 (6178 - six thousand one hundred and seventy-eight) 3. Count on and back in ones, tens, hundreds and thousands from four-digit numbers 4. Apply place value to partition, rearrange and regroup numbers to 10 000 into thousands, hundreds, tens and ones/units 5. Compare and order numbers up to four digits (using <, >, =) - focus on three- and four-digit numbers 6. Use negative numbers in context (temperature) 7. Investigate and use the properties of odd and even numbers, e.g. odd+odd=even, even+even=even 8. Read, write, compare and order ordinal numbers and use them in real-life situations 9. Use the language of fractions (fraction, numerator, denominator, decimal, decimal point) 10. Recognise that the place value system can be extended to tenths 11. Read, write, compare and order decimals to tenths and locate and represent them on a number line (0.1-9.9) 12. Use decimal notation for tenths and hundredths in context (length, money) 13. Add and subtract decimals to hundredths in context (length, money) 14. Round numbers with one decimal place to the nearest whole number 15. Interpret the numerator and denominator of a fraction ($\frac{3}{8}$ means 3 equal parts of 8) 16. Read, write, compare and order commonly used fractions (halves, thirds, quarters, fifths, eighths and tenths) and locate and represent them on a number line (compare and order fractions with the same denominator including mixed numbers, e.g. $\frac{1}{4}$ __ $\frac{3}{4}$, 1 $\frac{1}{5}$ __ 1 $\frac{3}{5}$) 17. Count by quarters, halves and thirds including with mixed numbers ($\frac{1}{3}$, $\frac{2}{3}$, $\frac{3}{3}$, $\frac{4}{3}$, ... or $\frac{1}{3}$, $\frac{2}{3}$, 1, 1 $\frac{1}{3}$, ...) 18. Investigate equivalent fractions: $\frac{1}{2} = \frac{2}{4} = \frac{4}{8} = \frac{5}{10}$; $\frac{1}{4} = \frac{2}{8}$; $\frac{1}{5} = \frac{2}{10}$ 19. Investigate fractions equivalent to one whole (1= $\frac{2}{2}$, three thirds=1, 1= $\frac{6}{6}$) 20. Find halves, thirds, quarters, fifths, eighths and tenths of shapes and numbers (whole number answers) 21. Make connections between fractions and decimal notation (understand the equivalence between one-place decimals and fractions in tenths; understand that $\frac{1}{2}$ is equivalent to 0.5 and also $\frac{5}{10}$; understand that $\frac{1}{4}$ is equivalent to 0.25) 22. Represent and solve addition and subtraction problems using increasingly efficient mental and written strategies for computation and appropriate digital technologies - add and subtract pairs of two-digit numbers; add a two-digit number and a three-digit number; subtract a two-digit number from a three-digit number; add and subtract pairs of three-digit numbers 23. Use such written strategies for addition and subtraction as written algorithm with trading and number lines (jump strategy)

24. Find multiples of 10, 100, 1000 more/less than numbers mentally (up to four digits), e.g. 3407+20=3427, 3567-300=3265
25. Derive quickly pairs of two-digit numbers with a total of 100, e.g. 72+_=100
26. Derive quickly pairs of multiples of 50 with a total of 1000, e.g. 450+_=1000
27. Derive quickly doubles of all whole numbers to 50 and halves of even numbers to 100
28. Add and subtract near multiples of 10 or 100 to or from three-digit numbers and adjust, e.g. 367-198, 278+49
29. Recognise the result of multiplication as product
30. Use efficient mental and written strategies and appropriate digital technologies for multiplication and division - double any two-digit number; multiply multiples of 10 to 90 by a single-digit number; multiply a two-digit number by a single-digit number; divide a two-digit number by a single-digit number (answers no greater than 20)
31. Solve simple division problems with remainders (leftovers); decide whether to round up or down after division to give an answer to a problem, e.g. 25÷6=4r.1, 13÷3=4r.1
32. Record multiplication algorithms horizontally and vertically (in columns)
33. Recall quickly multiplication facts up to 10 × 10 including zero and related division facts
34. Check multiplication, e.g. 6x8=48 by doing 6x4 and doubling
35. Identify multiples of 3, 4, 6, 7, 8 and 9 up to the 10th multiple
36. Recognise multiples of 5, 10 and 100 up to 1000
37. Understand the effect of multiplying and dividing three-digit numbers by 10 (whole number answers)
38. Use estimation and rounding to check the reasonableness of answers to calculations (round three- and four-digit numbers to the nearest 10 or 100)
39. Recognise, describe and order Polish (PLN) and European (EUR) coins and notes according to their value (consolidate using money notation)
40. Solve problems involving purchases and the calculation of change to the nearest five grosz or cents with and without digital technologies

Grade 4 learning outcomes

Data handling	
Conceptual understandings: Probability can be based on experimental events in daily life (P3) Probability can be expressed in numerical notations (P3) Different graph forms highlight different aspects of data more efficiently (P3)	**Learners will develop an understanding that:** - probability is based on experimental events - scale can represent different quantities in graphs - the mode can be used to summarize a set of data - different types of graphs have special purposes

Probability

1. Describe the likelihood of an event by considering the number of possible outcomes
2. Express probability using simple numerical notations, e.g. 1 out of 6, 1 in 4
3. Conduct chance experiments with a small number of trials, e.g. throwing a die, predict and list their possible outcomes, identify and compare variation in their results

Organising, categorising and representing data

4. Pose questions and collect categorical or numerical data by observation or survey
5. Construct and interpret displays appropriate for data type including diagrams, tally marks and frequency tables, bar graphs (with different intervals of vertical axis) and line graphs
6. Compare the impact of data representations in bar graphs where scales have different intervals on the vertical axis
7. Compare line graphs with bar graphs (analysing where intermediate points on a graph have and do not have meaning)
8. Identify, describe and explain range and scale (vertical and horizontal axis)
9. Find and interpret range (lowest and highest value) and mode (most frequent answer)

Grade 4 learning outcomes

Measurement	
Conceptual understandings: Standard units allow us to have a common language to identify, compare, order and sequence objects and events (P2) Objects and events have attributes that can be measured using appropriate tools (P3) Conversion of units and measurements allows us to make sense of the world we live in (P4)	**Learners will develop an understanding that:** - the use of standard units to measure perimeter, area and volume - measures can fall between numbers on a measurement scale, for example, 3½ kg, between 4 and 5 cm - the unit conversions within measurement system (metric or customary)

Length, mass and capacity

1. Estimate, measure, compare and record lengths, masses and capacities using standard units (km, m, cm, mm/kg, g/l, ml) - consolidation
2. Convert between common metric units of capacity, mass and length (ml and l/g and kg/mm, cm, m and km) using decimals to one place, e.g. 2.6kg=2600g
3. Use decimal and fractional notation in measurement, e.g. 3.2 cm, 1.4 kg, 1 ½ m using decimals to one place
4. Draw and measure lines to the nearest centimetre and millimetre
5. Select and use appropriate units of measurement and tools to solve problems in real-life situations involving length, mass, and capacity

Area and perimeter

6. Measure and calculate the perimeter of regular and irregular polygons using familiar metric units
7. Understand that area is measured in square units (m^2, cm^2)
8. Find and use the formula for the perimeter and area of a rectangle (the relationship between perimeter and area)
9. Select and use appropriate units of measurement and tools to solve problems in real-life situations involving perimeter and area

Time

10. Read and tell digital and analogue time on 12-hour and 24-hour clocks; compare 12- and 24-hour systems and convert between them
11. Calculate time intervals in seconds, minutes and hours using digital or analogue formats, in days, weeks, months and years using a calendar
12. Convert between common metric units of time (sec, min, h, day, week, month, year)
13. Use measures of time and tools (timelines, timetables, calendars) to assist with problem solving in real-life situations

Grade 4 learning outcomes

Shape and space	
Conceptual understandings: Changing the position of a shape does not alter its properties (P3) Geometric shapes and vocabulary are useful for representing and describing objects and events in real-world situations (P3) Manipulation of shape and space takes place for a particular reason (P4)	**Learners will develop an understanding that:** - the common language used to describe shapes - the properties of regular and irregular polygons - the properties of regular and irregular polyhedra - congruent or similar shapes - lines and axes of reflective and rotational symmetry assist with the construction of shapes - 2D representations of 3D objects can be used to visualize and solve problems - an angle as a measure of rotation - directions for location can be represented by coordinates on a grid

Shape and geometric reasoning

1. Consolidate the ability to describe polygons
2. Draw different polygons without given properties
3. Draw rectangles and squares on a grid paper with given properties using a set square
4. Identify and describe properties of triangles and classify them as right-angled, equilateral, isosceles and scalene triangles
5. Identify and draw parallel, perpendicular and intersecting lines; recognise them in 2D shapes, drawings and the environment
6. Describe and compare lines, rays and segments
7. Describe and model congruency (the same shape and size) and similarity (the same shape, different size) in 2D shapes
8. Classify, compare and describe key properties of 3D shapes (pyramids, prisms, cones, cylinders and spheres)
9. Connect 3D shapes with their nets and other 2D representations
10. Recognise cross-sections of 3D shapes
11. Identify lines and axes of reflective and rotational symmetry and create symmetrical patterns including tessellation of regular polygons and patterns with two lines of symmetry
12. Classify angles as right, acute and obtuse, straight and draw them with the use of a protractor
13. Identify the arms and vertex of an angle
14. Measure angles to the nearest 5^{0} using a protractor (up to 180^{0})

Position and movement

15. Describe translations, reflections and rotations of 2D shapes (slide, flip, turn) and use those transformations to solve problems
16. Use simple maps and grid reference system to represent position, describe and follow routes
17. Describe the location of an object on a map using N, S, E, W, NE, NW, SE, SW - consolidation
18. Read and plot coordinates in the first quadrant

Grade 4 learning outcomes

Pattern and function	
Conceptual understandings: Functions are relationships or rules that uniquely associate members of one set with members of another set (P3) Patterns can often be generalized using algebraic expressions, equations or functions (P4)	**Learners will develop an understanding that:** - patterns can be generalized by a rule - patterns can be represented, analysed and generalized using tables, graphs, words, and, when possible, symbolic rules - the inverse relationship between addition and subtraction - the associative and commutative properties of addition - multiplication is repeated addition and that division is repeated subtraction - the inverse relationship between multiplication and division - the associative and commutative properties of multiplication

1. Describe, extend and create number patterns resulting from all four operations
2. Describe, extend and create patterns with fractions and decimals
3. Describe and represent the rules for patterns in a variety of ways (using words, charts and symbols)
4. Complete equivalent number sentences by calculating missing values (all four operations)
5. Identify and use the order of operations (without the use of brackets)
6. Use the commutative and associative properties of addition and multiplication and the relationships between operations (addition-subtraction, addition-multiplication, subtraction-division, multiplication-division) to check the results and solve problems
7. Solve single and multi-step word problems including all four operations and represent them using e.g. diagrams, number lines

Grade 4 learning outcomes

Number
Conceptual understandings: The base 10 place value system can be extended to represent magnitude (P3) Fractions, decimal fractions and percentages are ways of representing whole-part relationships (P4) For fractional and decimal computation, the ideas developed for whole-number computation can apply (P4) The operations of addition, subtraction, multiplication and division are related to each other and are used to process information to solve problems (P2-3) Even complex operations can be modelled in a variety of ways, for example, an algorithm is a way to represent operation (P3)
1. Read and write numbers up to one million 2. Write the number names to 1 000 000 (366178 - three hundred sixty-six thousand one hundred and seventy-eight) 3. Count on and back in steps of constant size, extending beyond zero 4. Apply place value to partition, rearrange and regroup numbers up to one million into thousands, hundreds, tens and ones/units 5. Compare and order numbers up to six digits (<, >, =) - focus on five- and six-digit numbers 6. Compare and order negative numbers on a number line or temperature scale; calculate a rise or fall in temperature mentally 7. Identify prime and composite numbers (introduce the term and list examples) 8. Identify square and cube numbers (introduce the term and list examples) 9. Calculate square and cube power of natural numbers 10. Read and write numbers using Roman numerals up to 3000 11. Use the language of fractions (fraction, numerator, denominator, decimal, decimal point, equivalence, mixed number) 12. Recognise that the place value system can be extended to hundredths 13. Use decimal notation for tenths and hundredths and understand what each digit represents 14. Read, write, compare and order decimals to hundredths and locate and represent them on a number line (compare and order decimals with the same number of places) 15. Add and subtract decimals to hundredths with the same number of decimal places 16. Round numbers with one or two decimal places to the nearest tenth or to the nearest whole number 17. Interpret mixed numbers (whole number and fraction, eg. $1\frac{1}{3}$) and locate and represent mixed numbers on a number line 18. Read and write fractions and locate and represent them on a number line 19. Write fractions using words, e.g one fifth 20. Compare and order fractions with the same and related denominators including mixed numbers, e.g. $\frac{1}{3}$ ___ $\frac{5}{6}$ 21. Recognise equivalence of fractions (halves-tenths), e.g. $\frac{1}{2} = \frac{5}{10}$; $\frac{1}{3} = \frac{3}{9}$ 22. Make one whole (halves-tenths), e.g. $\frac{1}{3}$ + _ = 1 23. Relate finding fractions to division (recognise that a division symbol equals a fraction line) 24. Find simple fractions of quantities, e.g. $\frac{3}{4}$ of 20 25. Use fractions to describe simple proportions, e.g. $\frac{1}{3}$ of the beads are yellow 26. Add and subtract fractions with the same denominator 27. Understand percentage as the number of parts in every 100 (introduce the term and symbol)

28. Find simple percentages of quantities (25%, 50%, 75%, 100%)
29. Make connections between equivalent fractions, decimals and percentages
30. Express halves, tenths, hundredths as percentages
31. Convert fractions into decimals (halves, quarters, tenths and hundredths)
32. Convert decimals into fractions and simplify, e.g. 2.5 = $2\frac{5}{10} = 2\frac{1}{2}$
33. Use fractions, decimals and percentages in real-life situations
34. Solve problems involving addition and subtraction using efficient mental and written strategies for computation and appropriate digital technologies - find the total of more than three two- or three-digit numbers; add or subtract any pair of three- and/or four-digit numbers
35. Use such written strategies for addition and subtraction as written algorithm with trading, number lines (jump strategy), split strategy
36. Know by heart pairs of one-place and two-place decimals with a total of 1, e.g. 0.8+0.2=1, 0.78+0.22
37. Derive quickly pairs of one-place decimals with a total of 10, e.g. 7.8+2.2
38. Derive quickly doubles of any number up to 100 and halves of even numbers to 200
39. Double and halve numbers with one (or two) decimal places, e.g. double 3.4, half of 8.6
40. Add or subtract near multiples of 10 or 100 to or from three- and four-digit numbers and adjust, e.g. 4387-299 (compensation strategy)
41. Recognise the elements of multiplication: factor x factor = product
42. Solve problems involving multiplication and division, including those that result in a remainder, using efficient mental and written strategies and appropriate digital technologies - multiply three-digit numbers by single-digit numbers; multiply pairs of two-digit numbers; multiply two-digit numbers with one decimal place by single-digit numbers, e.g. 3.6x7; divide three-digit numbers by single-digit numbers without remainders; divide three-digit numbers by single-digit numbers including those with a remainder (answers no greater than 30)
43. Express remainders as a fraction of the divisor when dividing two-digit numbers by a single-digit number, e.g. $23 \div 4 = 5\text{r}.3 = 5\frac{3}{4}$
44. Use written algorithm for multiplication and division (column strategy)
45. Automatically recall multiplication facts up to 10 × 10 including zero and related division facts (consolidation)
46. Identify and describe factors of two-digit numbers and multiples of any single-digit number (to the 10th multiple)
47. Recognise multiples of 5, 10, 25, 50 and 100 up to 1000
48. Find a missing digit in a number to make it a multiple of 2, 5, 10 and 100, e.g. 2_ is a multiple of 5
49. Know and apply tests of divisibility by 2, 5, 10 and 100
50. Understand the effect of multiplying and dividing any number up to 10 000 by 10 or 100 (whole number answers)
51. Multiply and divide mentally numbers ending with 0
52. Multiply by 19 or 21 by multiplying by 20 and adjusting, e.g. 35x19=(35x20)-19
53. Use estimation and rounding to check the reasonableness of answers to calculations (round four-digit numbers to the nearest 10, 100, 1000)
54. Solve problems involving purchases and the calculation of change in PLN and EUR with and without digital technologies

Grade 5 learning outcomes

Data handling	
Conceptual understandings: Probability can be expressed in numerical notations (P3) Probability can be represented on a scale between 0-1 or 0%-100% (P4) The probability of an event can be predicted theoretically (P4) Data can be presented effectively for valid interpretation and communication (P4) Range, mode, median and mean can be used to analyse statistical data (P4)	**Learners will develop an understanding that:** - probability can be expressed in scale (0-1) or per cent (0%-100%) - there is the difference between experimental and theoretical probability - different types of graphs have special purposes - mode, median, mean and range can summarize a set of data - one of the purposes of a database is to answer questions and/or solve problems

Probability

1. Express probability using numerical notations (fractions, decimals) and scale (0–1) or percent (0%-100%)
2. Conduct chance experiments with both small and large numbers of trials
3. Compare observed frequencies across experiments (experimental probability) with expected frequencies (theoretical probability) and explain the difference

Organising, categorising and representing data

4. Create, interpret and compare a range of data displays, including different tables, graphs, charts and diagrams (e.g. line graphs, circle graphs (pie charts), side-by-side column/bar graphs for two categorical variables)
5. Analyse statistical data using range (lowest and highest value), mode (most frequent answer), median (middle value) and mean (average) and explore how statistics are used in everyday life
6. Create and manipulate an electronic database for own purposes, including setting up spreadsheets and using simple formulas to create graphs

Grade 5 learning outcomes

Measurement	
Conceptual understandings: Standard units allow us to have a common language to identify, compare, order and sequence objects and events (P2) Accuracy of measurement depends on the situation and the precision of the tool (P4) Conversion of units and measurements allows us to make sense of the world we live in (P4) A range of procedures exists to measure different attributes of objects and events (P4)	**Learners will develop an understanding that:** - the use of standard units to measure perimeter, area and volume - the unit conversions within measurement system (metric or customary) - the procedures for finding area, perimeter and volume - the relationship between area and perimeter, between area and volume, and between volume and capacity - an angle as a measure of rotation

Length, mass and capacity

1. Select and use standard units of measurement for length, mass and capacity (km, m, cm, mm/kg, g/l, ml); read and write measurements using decimals to two or three places
2. Convert between common metric units of capacity, mass, length (ml and l/g and kg/mm, cm, m and km), using decimals to three places, e.g. 1.245m=1m 24.5cm
3. Use decimal and fractional notation in measurement, for example, 3.2 cm, 1.47 kg, 1 ½ m, using decimals to three places
4. Draw and measure lines to the nearest centimetre and millimetre - consolidation
5. Select and use appropriate units of measurement and tools to solve problems in real-life situations involving length, mass, and capacity

Area, perimeter and volume

6. Measure and calculate the perimeter and area of squares, rectangles and simple compound shapes that can be split into rectangles
7. Calculate the perimeter and area of triangles
8. Calculate the perimeter of triangles when given the correlation between the sides, e.g. side b is twice as long as side a
9. Calculate the length of sides of an equilateral triangle when given its perimeter
10. Calculate the area of a right-angled triangle when given its legs
11. Calculate the area of trapeziums, rhombi and parallelograms
12. Calculate the length of one side of a rectangle when given its area and the length of the other side
13. Understand that volume is measured in cubic units (m^3, cm^3)
14. Develop and describe formulas for finding the volume of cubes and rectangular prisms
15. Connect volume and capacity and their units of measurement (the relationship between volume and capacity)
16. Select and use appropriate units of measurement and tools to solve problems in real-life situations involving perimeter, area and volume

Time

17. Read and tell digital and analogue time on 12-hour and 24-hour clocks; compare 12- and 24-hour systems and convert between them - consolidation
18. Convert between common metric units of time (sec, min, h, day, week, month, year, decade, century)
19. Determine time worldwide and compare various time zones
20. Use measures of time and tools (timelines, calendars, schedules, timetables) to assist with problem solving in real-life situations

Grade 5 learning outcomes

Shape and space	
Conceptual understandings: Manipulation of shape and space takes place for a particular reason (P4) Consolidating what we know of geometric concepts allows us to make sense of and interact with our world (P4) Geometric tools and methods can be used to solve problems relating to shape and space (P4)	**Learners will develop an understanding that:** - the common language used to describe shapes - the properties of regular and irregular polygons - the properties of regular and irregular polyhedra - the properties of circles - scale (ratios) is used to enlarge and reduce shapes - systems for describing position and direction - 2D representations of 3D objects can be used to visualize and solve problems - visualization of shape and space is a strategy for solving problems - geometric ideas and relationships can be used to solve problems in other areas of mathematics and in real life

Shapes and geometric reasoning

1. Introduce the term polygon
2. Classify different regular and irregular polygons and understand whether a 2D shape is a polygon or not
3. Identify and describe the properties of quadrilaterals (squares, rectangles, parallelograms, rhombi and trapeziums), and classify them using parallel sides, equal sides, equal angles, etc.
4. Identify and draw diagonals on 2D shapes
5. Recognise types of trapeziums, e.g. right-angled, isosceles
6. Recognise types of triangles according to their sides and angles
7. Recognise heights, bases and arms of triangles
8. Draw triangles using a compass and a ruler
9. Use the triangle inequality rule - a+b>c, where c is the longest side
10. Identify parts of a circle (centre, radius, diameter, circumference, sector, semicircle and quadrant)
11. Differentiate between a disk (all points inside the circle) and a circle (circumference)
12. Draw trapeziums, triangles, circles and disks with given properties
13. Make enlargements and reductions of 2D pictures using scale (ratio)
14. Describe and sketch 3D shapes (pyramids, prisms, cylinders, cones and spheres)
15. Draw nets of simple 3D shapes
16. Describe and compare lines, rays and segments and draw them with precision to 1mm
17. Draw segments in a given scale
18. Recognise and show relation between a point, segment, ray and line
19. Indicate points that belong to a given line and segment
20. Classify angles as acute, right, obtuse, straight, reflex or revolution; recognise vertical and adjacent angle pairs
21. Measure and draw angles to the nearest degree using a protractor up to 360°

22. Compare angles in different 2D shapes; check that the sum of the angles in a triangle is 180° and in a quadrilateral - 360°

Position and movement

23. Describe routes using landmarks and directional language
24. Find a place on a grid/map given its coordinates (including Cartesian coordinate system)
25. Use scale to calculate the distance between two points on a map
26. Read and plot coordinates in all four quadrants
27. Solve problems relating to shape and space using geometric tools and methods

Grade 5 learning outcomes

Pattern and function
Conceptual understandings: Patterns can often be generalized using algebraic expressions, equations or functions (P4) Exponential notation is a powerful way to express repeated products of the same number (P4) **Learners will develop an understanding that:** - patterns can be generalized by a rule - patterns can be represented, analysed and generalized using tables, graphs, words, and, when possible, symbolic rules - the exponents as repeated multiplication - the inverse relationship between exponents and roots
1. Describe, extend and create patterns involving whole numbers, fractions and decimals using tables, graphs, words, and, when possible, symbolic rules 2. Describe the rules used to create the pattern 3. Complete simple equations including using letters to represent the quantity (5+y=24, ½ of x=5) 4. Explore the use of brackets and order of operations to write number sentences 5. Use the relationships between operations to check the results and solve problems 6. Identify and model exponents as repeated multiplication (3^2=3x3, 4^3=4x4x4) 7. Identify and model the inverse relationship between exponents and roots ($3^2=\sqrt{9}$) 8. Make sense of and solve word problems, single and multi-step, applying the understanding of patterns and the four operations - use brackets to show the series of calculations necessary

Grade 5 learning outcomes

Number
Conceptual understandings: The base 10 place value system extends infinitely in two directions (P4) Fractions, decimal fractions and percentages are ways of representing whole-part relationships (P4) For fractional and decimal computation, the ideas developed for whole-number computation can apply (P4) The operations of addition, subtraction, multiplication and division are related to each other and are used to process information to solve problems (P2-3) Even complex operations can be modelled in a variety of ways, for example, an algorithm is a way to represent operation (P3)
1. Read and write whole numbers up to millions 2. Write the number names to millions 3. Count on and back in fractions and decimals, e.g. ⅓s, 0.1s, and repeated steps of whole numbers also through zero 4. Apply place value to partition, rearrange and regroup numbers to millions, and to thousandths 5. Compare and order positive numbers to millions 6. Read and write negative integers, locate and represent them on a number line and investigate everyday situations that use them 7. Compare and order negative integers to -30 with and without the use of the number line 8. Find the difference between a positive and negative integer, and between two negative integers in a context such as temperature or on a number line 9. Identify and describe the properties of prime and composite numbers; know prime numbers up to 20 and find all prime numbers less than 100 10. Identify, read and write square and cubed numbers (exponents) as well as square roots 11. Recognise the historical origins of our number system and begin to understand how it developed 12. Use the language of fractions (fraction, numerator, denominator, decimal, decimal point, equivalence, mixed number, improper fraction) 13. Recognise that the place value system can be extended to thousandths 14. Read, write, compare and order decimals to thousandths (compare and order decimals with the same and different number of places, e.g. 2.72>2.7) 15. Add and subtract decimals to thousandths with the same or different number of decimal places, with and without digital technologies 16. Multiply and divide decimals by numbers ending with 0 (answers up to two decimal places for division) 17. Multiply decimals by whole numbers, with and without digital technologies, including using the written algorithm 18. Divide decimals by whole numbers (where the results are terminating decimals) with and without digital technologies 19. Round numbers with three decimal places to the nearest hundredth, tenth or to the nearest whole number 20. Read and write fractions including mixed numbers, and locate and represent them on a number line 21. Compare and order fractions with related and unrelated denominators and with the same numerator 22. Simplify fractions in mental and written form (where their simplest form is $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, or the number of fifths or tenths) 23. Convert improper fractions to mixed numbers and vice versa 24. Relate finding fractions to division and use them as operators to find fractions of quantities, e.g. $\frac{7}{10}$ of 40 25. Calculate fractions raised to the second and third power 26. Add and subtract fractions with the same and related denominators 27. Multiply fractions and mixed numbers by whole numbers using cross-cancelling 28. Find the inverse of fractions including mixed numbers and whole numbers, e.g. the inverse of $\frac{3}{4}$ is $\frac{4}{3}$, the inverse of 2 is $\frac{1}{2}$

29. Divide fractions and mixed numbers using cross-cancelling
30. Solve simple problems involving ratio and direct proportion
31. Find simple percentages of whole numbers, e.g. find discounted prices
32. Make connections and convert between equivalent fractions, decimals and percentages (recognise equivalence between decimal and fraction forms; begin to convert a vulgar fraction to a decimal fraction using division; express halves, quarters, thirds, tenths, hundredths as percentages)
33. Use fractions, decimals and percentages interchangeably in real-life situations
34. Select and apply efficient mental and written strategies and appropriate digital technologies to solve problems involving addition and subtraction within million - numbers with the same or different number of digits/decimal places
35. Use such written strategies for addition and subtraction as written algorithm with trading, number lines (jump strategy), split strategy, compensation strategy
36. Use place value and known number facts to add or subtract two-digit whole numbers mentally
37. Derive quickly doubles of any two-digit number, e.g. 78, 7.8, 0.78 and derive corresponding halves
38. Add or subtract near multiples of one when adding numbers with one decimal place and adjust, e.g. 5.6+2.9, 13.5-2.1 (compensation strategy)
39. Add or subtract near multiples of 10, 100 or 1000 and adjust, e.g. 4387-2999, 3127+4997
40. Recognise the result of multiplication as product and its elements as factors (revision)
41. Recognize the elements of division: dividend ÷ divisor = quotient
42. Find a missing factor in multiplication and a missing dividend or divisor in division
43. Select and apply efficient mental and written strategies and appropriate digital technologies to solve problems involving multiplication and division - multiply two-, three- or four-digit numbers by a single-digit number or by two-digit numbers; divide three-digit numbers by single-digit numbers, including those leaving a remainder; divide three-digit numbers by two-digit numbers with no remainder
44. Give an answer to division as a mixed number and a decimal (with divisors of 2, 4, 5, 10 or 100)
45. Use written algorithm for multiplication and division (including the Polish version of the written algorithm as an example)
46. Use written algorithm to multiply by numbers ending with 0
47. Automatically recall multiplication facts up to 12 x 12 including zero and related division facts
48. Use known number facts to generate new multiplication facts, e.g. the 17x table from 10x + 7x tables
49. Identify and describe factors and multiples of a given number (consolidation)
50. Recognise divisibility of numbers by 2, 3, 4, 5, 8, 9, 10, 25 and 100
51. Understand the effect of multiplying and dividing any number by 10 or 100 (answers including decimals), e.g. 27÷100
52. Multiply near multiples of 10 (19, 41, 59, 61, etc.) by multiplying by 10, 20, 30, etc. and adjusting, e.g. 35x41=(35x40)+35
53. Multiply by halving one number and doubling the other, e.g. 35x16=70x8
54. Use place value and known multiplication facts to multiply/divide mentally, e.g. 0.8x7; 4.8÷6
55. Divide two-digit numbers by single-digit numbers mentally, including leaving a remainder
56. Use estimation and rounding to check the reasonableness of answers to calculations (round whole numbers to the nearest 10, 100, 1000)
57. Solve problems involving purchases and the calculation of change in PLN and EUR with and without digital technologies